Name of college - S. S. College, J-Bad

Deptt - Mathematics

Topic - oblique Asymptotes of Algebraic Curve

class - B.Sc I (Hons)

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Study Material

(1) If the Equation of a curve of the nth degree be thrown in to the form

$$(ax+by+c)P_{n-1} + F_{n-1} = 0 \text{ Then}$$

the Equation

$$ax+by+c + \lim_{y \to -b/a,\, x=\infty} \frac{F_{n-1}}{P_{n-1}} = 0$$

Ex:→

(2) If the Equation of the curve of nth degree can be put into the form

$$[ax+by+c]^2 P_{n-2} + F_{n-2} = 0 \text{ the}$$

the Equations

Now for c

$$c = -\frac{\phi_2(m)}{\phi_3'(m)}$$

Here $\phi_2(m) = -3am$

$\phi_3(m) = 1+m^3$

$\phi_3'(m) = 3m^2$

$$= -\frac{-3am}{3m^2}$$

$$= +\frac{a}{m}$$ put $m=-1$

$$= -a$$

real Asymptote

$\therefore$ $y = mx + c$

$= -x - a$

$\therefore$ $x + y + a = 0$ is only one real Asymptote.

Since other two values of m are imaginary therefore the two asymptotes corresponding to these values of m are imaginary.

Problem 2. Find the Asymptotes of

$$(x^2-y^2)^2 = 2(x^2+y^2)$$

Solution $\Rightarrow$ Here Equation of Curve is

$$(x^2-y^2)^2 = 2(x^2+y^2)$$

$$\Rightarrow x^4 - 2x^2y^2 + y^4 - 2x^2 - 2y^2 = 0$$

$$\phi_4(x,y) = x^4 - 2x^2y^2 + y^4$$

$$\phi_2(x,y) = -2x^2 - 2y^2$$

Put $x=1$ and $y=m$

$$f_4(m) = 1 - 2m^2 + m^4$$
$$= (m^2-1)^2$$

$$f_3(m) = 0$$
$$f_2(m) = -2 - 2m^2$$
$$= -2(1+m^2)$$

$$f_4(m) = 0 \Rightarrow (m^2-1)^2 = 0$$
$$\Rightarrow m^2 - 1 = 0$$
$$\Rightarrow m = +1, -1, +1, -1$$

Here the values of m obtained by $f_4(m) = 0$ repeated twice.

for c,

$$\frac{c^2}{2} f_4''(m) + c f_3'(m) + f_2(m) = 0 \quad \text{— (1)}$$

$$f_4(m) = (m^2-1)^2 \Rightarrow f_4'(m) = 2(m^2-1).2m$$
$$= 4m^3 - 4m$$
$$f_4''(m) = 12m^2 - 4$$

Putting these in (1)

$$\frac{c^2}{2}(12m^2-4) + 0 + \{-2(1+m^2)\} = 0$$

$$\frac{c^2}{2} \times 4(3m^2-1) - 2(1+m^2) = 0$$

When $m = 1$

$$\frac{c^2}{\cancel{2}}(\overset{2}{\cancel{4}}(3-1)) - 2(1+1) = 0$$
$$2c^2 \times 2 - 4 = 0$$

$\Rightarrow 4C_0^2 = 4 \quad \Rightarrow C_0 = \pm 1$

for Asymptotes $\quad C_0 = 1, \; m = 1 \quad m = -1$
$\quad C_0 = -1 \quad m = -1 \quad m = -1$

$y = mx + C \Rightarrow y = x + 1 \qquad C = 1, m = 1$
$\Rightarrow x - y + 1 = 0 \qquad C = -1 \quad m = 1$
$y = x - 1$
$x - y - 1 = 0 \qquad C = -1 \quad m = 1$
$y = -x + 1$
$x + y + 1 = 0 \qquad C = -1 \quad m = 1$
$y = -x - 1$
$x + y - 1 = 0$

Problem:→ Find all the Asymptotes of the Curve.

$$(x+y)^2 (x + 2y + 2) = x + 9y - 2$$

Sol^n → Here Equation of the Given Curve

$$(x+y)^2 (x + 2y + 2) = x + 9y - 2$$

$$\Rightarrow [x^2 + 2xy + y^2][x + 2y + 2] = [x + 9y - 2]$$

$$\Rightarrow x^3 + 2x^2y + 2x^2 + 2x^2y + 2xy^2 + 4xy + xy^2 + 2y^3 + 2y^2 - x - 9y + 2 = 0$$

$$\Rightarrow x^3 + 4x^2y + 3xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 2 = 0$$

$$q_3(x,y) = x^3 + 4x^2y + 3xy^2 + 2y^3$$
$$q_2(x,y) = 2x^2 + 4xy + 2y^2$$
$$q_1(x,y) = -x - 9y$$

Put $x = 1$ $y = m$

$q_3(m) = (1+m)^2 (1+2m)$

$q_2(m) = 2(1+m)^2$

$q_1(m) = -(1+9m)$

for m, $q_3(m) = 0$

$\Rightarrow (1+m)^2 (1+2m) = 0$

$\Rightarrow m = -1, -1, \quad m = -1/2$

for Non repeated value of m,

for $m = -1/2$

$$c_0 = -\frac{q_2(m)}{q_3'(m)}$$

$$= -\frac{2(1+m)^2}{2(1+m)(1+2m) + 2(1+m)^2} =$$

$$= -\frac{1+m}{1+2m+1+m}$$

$$= -\frac{1+m}{2+3m} \quad \text{put } m = -1$$

$$= -\frac{1-1/2}{2-3/2} = -1$$

for repeated values of m

i.e for $m = -1, -1$

c is Given by.

$$\frac{1}{2}c^2\phi_3''(m)+c\,\phi_2'(m)+\phi_1(m)=0.$$

$$\Rightarrow \frac{1}{2}c^2\,2\left[2+3m\right]+3(1+m)\Big]+c(1+m)-(1-9m)=0$$

$\Rightarrow$ for $m=-1$

$$\frac{1}{2}c^2 2(2-3)+3(1-1)+c(1-1)-(1+9)=0$$

$$-c^2+80=0 \Rightarrow c=\pm 2\sqrt{2}.$$

hence the three Asymptotes are

$y=-\frac{1}{2}x-1$ $\quad$ $y=-x+2\sqrt{2}$ $\quad$ $y=-x-2\sqrt{2}$

$\Rightarrow x+2y+1=0$ $\quad$ $x+y-2\sqrt{2}=0$ $\quad$ $x+y+2\sqrt{2}=0$